

Formulas for Distance, Midpoint, & Slope

Distance Formula: The distance formula is derived from the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$

1. If the Pythagorean Theorem is solved for c , then what would c equal?

$$C = \sqrt{a^2 + b^2}$$

2. If Segment AB has endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ then

$$AB = \sqrt{\underbrace{(x_2 - x_1)^2}_{a^2} + \underbrace{(y_2 - y_1)^2}_{b^2}}$$

3. Analyze the distance formula above and write a short response to the following questions.

a. What does the AB mean and what does it represent in terms of the Pythagorean Theorem?

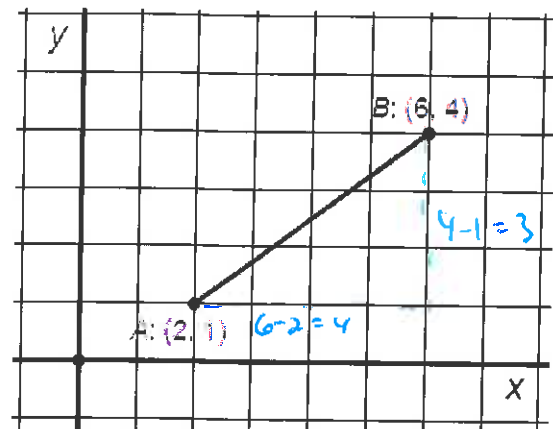
AB means the length of segment \overline{AB} . It takes the place of C in the Pythagorean Theorem.

b. What do the expressions $(x_2 - x_1)$ and $(y_2 - y_1)$ represent in terms of the Pythagorean Theorem?

$x_2 - x_1$ represents the "a" and $y_2 - y_1$ represents the "b"

4. Use the Distance Formula to find AB .

$$\begin{aligned} AB &= \sqrt{(6-2)^2 + (4-1)^2} \\ &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$



5. Segment PQ has endpoint $P(-5, 6)$ and $Q(2, -4)$. Find PQ to the nearest tenth.

$$\begin{aligned} PQ &= \sqrt{(-5-2)^2 + (6-(-4))^2} = \sqrt{(-7)^2 + (10)^2} = \sqrt{49 + 100} \\ &= \sqrt{149} \approx 12.2 \text{ units} \end{aligned}$$

Midpoint Formula

6. Is the following statement true or false? Explain why and support your claim with examples.

"The average of any two numbers will always be a value that is half way between the two numbers."

$$\frac{8+10}{2} = \frac{18}{2} = 9$$

9 is $\frac{1}{2}$ way
between 8 and 10.

$$\frac{15+17}{2} = \frac{32}{2} = 16$$

16 is $\frac{1}{2}$ between
15 and 17.

the statement
is
True!
←

7. If Segment AB has endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ then the midpoint, M, of segment AB is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Explain how the claim in #6 supports or justifies this formula.

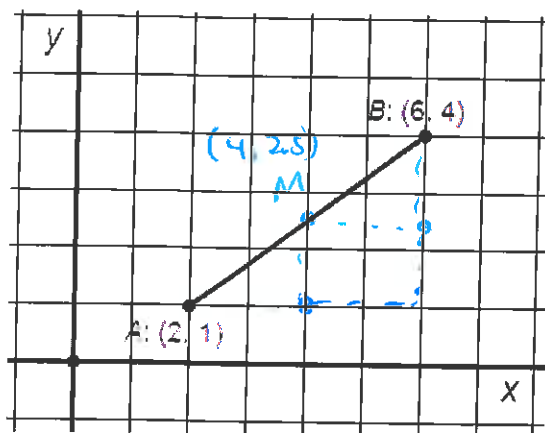
by averaging the x's, we can find the $\frac{1}{2}$ way point.
the same is true for the y values.

8. Use the Midpoint Formula to find the midpoint of \overline{AB} .

$$\frac{6+2}{2} = \frac{8}{2} = 4$$

$$\frac{4+1}{2} = \frac{5}{2} = 2.5$$

$$M(4, 2.5)$$



9. Segment PQ has endpoint $P(-5, 6)$ and $Q(2, -4)$. Find the midpoint of \overline{PQ} .

$$\frac{-5+2}{2} = \frac{-3}{2} = -1.5$$

$$\frac{6+(-4)}{2} = \frac{2}{2} = 1$$

$$M(-1.5, 1)$$

Slope Formula

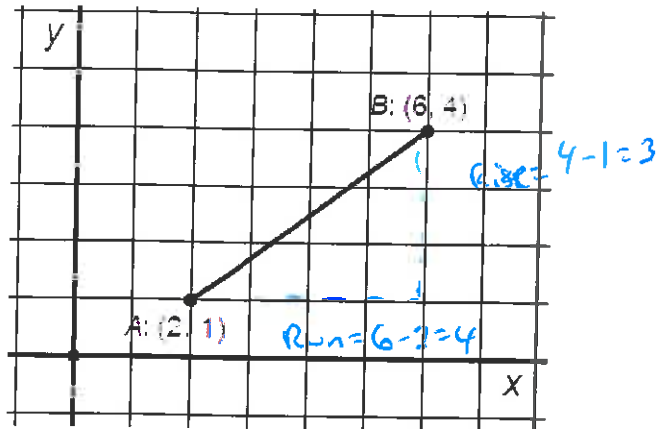
10. If Segment AB has endpoints A(x_1, y_1) and B(x_2, y_2) then the slope of segment AB is

$$m_{\overline{AB}} = \frac{y_2 - y_1}{x_2 - x_1} \quad \begin{array}{l} \text{Rise} \\ \text{Run} \end{array}$$

11. Use the Slope Formula to find the slope of \overline{AB} .

$$m_{\overline{AB}} = \frac{4-1}{6-2} = \frac{3}{4}$$

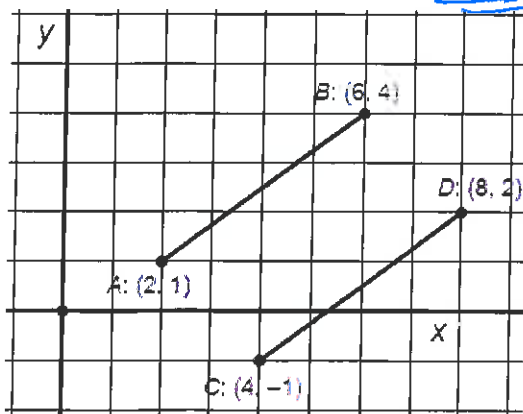
* Slope is always left as a fraction.



12. Segment PQ has endpoint P(-5, 6) and Q(2, -4). Find the slope of \overline{PQ} .

$$m_{\overline{PQ}} = \frac{6 - (-4)}{-5 - 2} = \frac{10}{-7}$$

13. In the picture, Segment AB is parallel to segment CD.



a. Find the slope of segment AB.

$$m = \frac{4-1}{6-2} = \frac{3}{4}$$

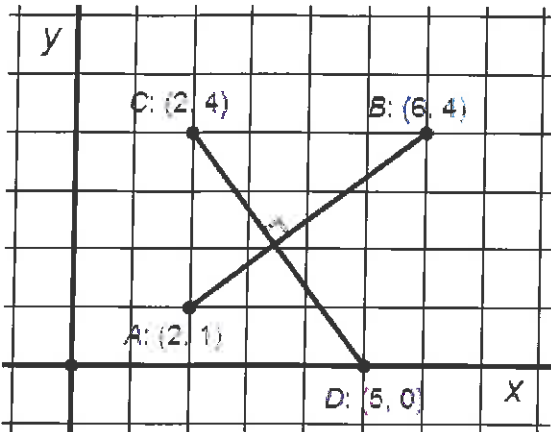
b. Find the slope of segment CD.

$$m = \frac{2-(-1)}{8-4} = \frac{3}{4}$$

c. What can be said of the slopes of parallel lines?

parallel lines have equal slopes.

14. In the picture, Segment AB is perpendicular to segment CD.



a. Find the slope of segment AB.

$$m = \frac{4-1}{6-2} = \frac{3}{4}$$

b. Find the slope of segment CD.

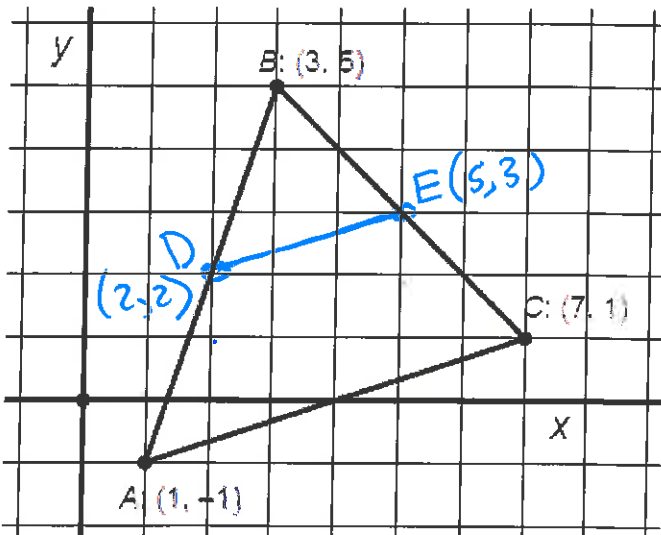
$$m = \frac{4-0}{2-5} = \frac{4}{-3}$$

c. What can be said of the slopes of perpendicular lines?

perpendicular lines have negative reciprocal slopes.

Putting it All Together

15. A Mid-segment of a triangle is a segment that connects the midpoints of 2 of the sides of the triangle.



a. Create Mid-segment \overline{DE} by finding and connecting the midpoints of sides \overline{AB} and \overline{CB} .

midpt of \overline{AB} :

$$\frac{3+1}{2} = \frac{4}{2} = 2$$

$$\frac{5+(-1)}{2} = \frac{4}{2} = 2$$

$$D(2, 2)$$

mp of \overline{CB} :

$$\frac{7+3}{2} = \frac{10}{2} = 5$$

$$\frac{1+5}{2} = \frac{6}{2} = 3$$

$$E(5, 3)$$

b. Find the slopes of Mid-segment \overline{DE} and side \overline{AC} .

What conclusion can you draw from your results?

$$\text{Slope } \overline{DE} = \frac{3-2}{5-2} = \frac{1}{3}$$

$$\text{Slope } \overline{AC} = \frac{1-(-1)}{7-1} = \frac{2}{6} = \frac{1}{3}$$

midsegment \overline{DE} is \parallel to side \overline{AC} .

c. Find DE and AC (remember this mean distance). What conclusion can you draw from your results?

$$\begin{aligned} DE &= \sqrt{(5-2)^2 + (3-2)^2} \\ &= \sqrt{3^2 + 1^2} \\ &= \sqrt{9+1} \\ &= \sqrt{10} \approx 3.16 \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(7-1)^2 + (1-(-1))^2} \\ &= \sqrt{6^2 + 2^2} \\ &= \sqrt{36+4} \\ &= \sqrt{40} \approx 6.32 \end{aligned}$$

\overline{AC} is twice as long as \overline{DE} .